Exercise 47

Find y' and y''.

$$y = \cos(\sin 3\theta)$$

Solution

Take the derivative using the chain rule.

$$\begin{aligned} y' = \frac{dy}{d\theta} &= \frac{d}{d\theta}\left[\cos(\sin 3\theta)\right] \\ &= [-\sin(\sin 3\theta)] \cdot \frac{d}{d\theta}(\sin 3\theta) \\ &= [-\sin(\sin 3\theta)] \cdot (\cos 3\theta) \cdot \frac{d}{d\theta}(3\theta) \\ &= [-\sin(\sin 3\theta)] \cdot (\cos 3\theta) \cdot (3) \\ &= -3\sin(\sin 3\theta)\cos 3\theta \end{aligned}$$

Take another derivative.

$$\begin{aligned} y'' = \frac{d}{d\theta}(y') &= \frac{d}{d\theta}[-3\sin(\sin 3\theta)\cos 3\theta] \\ &= -3\frac{d}{d\theta}[\sin(\sin 3\theta)\cos 3\theta] \\ &= -3\left\{\left[\frac{d}{d\theta}\sin(\sin 3\theta)\right]\cos 3\theta + \sin(\sin 3\theta)\left[\frac{d}{d\theta}(\cos 3\theta)\right]\right\} \\ &= -3\left\{\left[\cos(\sin 3\theta)\cdot\frac{d}{d\theta}(\sin 3\theta)\right]\cos 3\theta + \sin(\sin 3\theta)\left[(-\sin 3\theta)\cdot\frac{d}{d\theta}(3\theta)\right]\right\} \\ &= -3\left\{\left[\cos(\sin 3\theta)\cdot(\cos 3\theta)\cdot\frac{d}{d\theta}(3\theta)\right]\cos 3\theta + \sin(\sin 3\theta)\left[(-\sin 3\theta)\cdot(3)\right]\right\} \\ &= -3\left\{\left[\cos(\sin 3\theta)\cdot(\cos 3\theta)\cdot(3)\right]\cos 3\theta + \sin(\sin 3\theta)\left[(-\sin 3\theta)\cdot(3)\right]\right\} \\ &= -9[\cos(\sin 3\theta)\cos^2 3\theta - \sin(\sin 3\theta)\sin 3\theta] \end{aligned}$$